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Generalizations of the Chebyshev-type inequality for Choquet-like expectation

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ABSTRACT

This paper proves generalizations of the Chebyshev-type inequality for Choquet-like expectation. Some previous results are obtained as corollaries.

1. Introduction

Integral inequalities play important roles in classical probability and measure theory. We recall few famous classical inequalities, such as Hölder's, Minkowski's and Chebyshev's inequalities. In these inequalities rather often the additivity of the Riemann (Lebesgue) integral is substantially exploited. On the other hand, there are several non-additive integrals, such as the Choquet or Sugeno integrals, which have shown to be extremely useful in applications, especially when dealing with non-additive data in economics, sociology, psychology, multicriteria decision aid, etc.

The Choquet integral (also known as Choquet expectation) is a generalization of the Lebesgue integral, defined with respect to a non-classical measure, often called non-additive measure or also capacity. This integral was created by the French mathematician Gustave Choquet [12]. The Choquet integral shares several properties with the Lebesgue integral, including some inequalities. Recently, the study of integral inequalities has been extended to some more general classes of integrals [1,3,25,33].

Sugeno integral is an important kind of non-additive integrals which was introduced by Sugeno [36] and then was exploited by many authors [30,31,41]. Sugeno integral is a useful tool in several theoretical and applied statistics. For instance, in decision theory, the use of the Sugeno integral can be envisaged from two points of view: decision under uncertainty and multi-criteria decision-making [15]. Notice that the Sugeno integral is not an extension of the Lebesgue integral.

The study of inequalities for Sugeno integral was initiated by Román-Flores et al. [32], and then followed by the authors [1,2,18,26,27,29]. In [10], a Chebyshev type inequality for a special case for Sugeno integral was obtained which has been generalized by Ouyang et al. [27]. Furthermore, Chebyshev type inequalities for Sugeno integral were proposed in a rather general form by Mesiar et al. [2,26]. Later on, Girotto and Holzer [18] proved a characterization of comonotonicity property by a Chebyshev type inequality for Sugeno integral.

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The Choquet integral and the Sugeno integral have become widely used aggregation functions, especially in multicriteria decision making [15,20], subjective evaluation [21], information fusion [9,17,40], regression analysis [42], etc. In fact, the Sugeno and the Choquet integral contain all order statistics, thus in particular, min, max and the median [20,23].

The well-known Chebyshev type inequality is a part of the classical mathematical analysis (cf.[6,7,5,39,44]).

Definition 1.1 (*13,14,30*). Functions $u, v : X \to \mathbb{R}$ are said to be comonotone if for all $x, y \in X$,

$$(u(x) - u(y))(v(x) - v(y)) \geqslant 0.$$

The comonotonicity of functions u and v is equivalent to the non-existence of points $x, y \in X$ such that $u(x) < u(y)$ and $v(x) > v(y)$.

Given a measurable space $(X, \mathcal{F})$ and two $\mathcal{F}$-Borel measurable functions u,v, Armstrong [8] proved that, if u,v are Lebesgue integrable w.r.t. a probability measure P and are comonotone, then the following well known Chebyshev inequality

$$\int_X uvdP \geqslant \left(\int_X udP\right)\left(\int_X vdP\right),$$

holds (see also [19]).

The aim of this paper is to generalize the previous ones [1,19,26–28], and we believe that they will prove their usefulness in several areas, such as the economy and decision making, among others. The paper is organized as follows: Section 2 recalls the concepts of Choquet-like integrals while Section 3 presents our main results. Finally, some concluding remarks are added.

2. Preliminaries

In this section, we recall some basic definitions and previous results which will be used in the sequel. For details, we refer to [24]. For the convenience of the reader, we provide in this section a summary of the mathematical notations and definitions used in this paper (see [33]).

Definition 2.1 37. An operation $\oplus : [0, \infty]^2 \to [0, \infty]$ is called a pseudo-addition if the following properties are satisfied:

(P1) $a \oplus 0 = 0 \oplus a = a$ (neutral element);
(P2) $(a \oplus b) \oplus c = a \oplus (b \oplus c)$ (associativity);
(P3) $a \leqslant c$ and $b \leqslant d$ imply that $a \oplus b \leqslant c \oplus d$ (monotonicity);
(P4) $a_n \to a$ and $b_n \to b$ imply that $a_n \oplus b_n \to a \oplus b$ (continuity).

Definition 2.2. [24,37] Let $\oplus$ be a given pseudo-addition on [0,∞]. Another binary operation $\odot$ on [0,∞] is said to be a pseudo-multiplication corresponding to $\oplus$ if the following properties are satisfied:

(*M1*) $a \odot (x \oplus y) = (a \odot x) \oplus (a \odot y)$;
(*M2*) $a \leqslant b$ implies $(a \odot x) \leqslant (b \odot x)$;
(*M3*) $a \odot x = 0 \iff a = 0 \text{or} x = 0$;
(*M4*) $\exists\, e \in [0, \infty]$ (i.e., there exist the neutral element e) such that $e \odot x = x$ for any $x \in [0, \infty]$;
(*M5*) $a_n \to a \in (0, \infty)$ and $x_n \to x$ imply $(a_n \odot x_n) \to (a \odot x)$ and $\infty \odot x = \lim_{a\to\infty}(a \odot x)$;
(*M6*) $a \odot x = x \odot a$;
(*M7*) $(a \odot b) \odot c = a \odot (b \odot c)$.

Notice that a pseudo-multiplication $\odot$ corresponding to a given pseudo-addition $\oplus$ need to be neither commutative nor associative, in general. But in this paper, we will suppose that both (M6) and (M7) hold. Thus (M4) equivalents that there exists a neutral element e.

Mesiar [24] showed that, if $\odot$ is a pseudo-multiplication corresponding to a given pseudo-addition $\oplus$ fulfilling axioms (M1)-(M7) and if its identity element e is not an idempotent of $\oplus$, then there is a unique continuous strictly increasing function $g : [0, \infty] \to [0, \infty]$ with $g(0) = 0$ and $g(\infty) = \infty$, such that $g(e) = 1$ and

$$a \oplus b = g^{-1}(g(a) + g(b)) \quad \oplus \text{ is called a } g\text{-addition},$$

$$a \odot b = g^{-1}(g(a) \cdot g(b)) \quad \odot \text{ is called a } g\text{-multiplication}.$$

Mesiar [24] also proved that if the identity element e of the pseudo-multiplication is also an idempotent of $\oplus$ (i.e., $e \oplus e = e$), then $\oplus = \vee(=$ sup, i.e. the logical addition. In this case, the logical multiplication $\wedge$ and the g-multiplication are the candidates of $\odot$, among others.

Remark 2.3. Restricting to the interval [0,1] a pseudo-multiplication and a pseudo-addition with additional properties of associativity and commutativity can be considered as the t-norm T and the t-conorms S (see [22]), respectively.

Definition 2.4 23. A monotone measure μ on a measurable space $(X, \mathcal{F})$ is a function $\mu : \mathcal{F} \to [0, \infty]$ satisfying

(i) $\mu(\emptyset) = 0$;
(ii) $\mu(X) > 0$;
(iii) $\mu(A) \leqslant \mu(B)$ whenever $A \subseteq B$;

moreover, μ is called real if $\| \mu \| = \mu(X) < \infty$ and μ is said to be an additive measure if $\mu(A \cup B) = \mu(A) + \mu(B)$, whenever $A \cap B = \emptyset$. The triple $(X, \mathcal{F}, \mu)$ is also called a *monotone measure space* if μ is a monotone measure on $\mathcal{F}$.

Note that a monotone measure is not necessarily σ-additive. This concept goes back to M. Sugeno [36] (where also the continuity of the measures was required). To be precise, normed monotone measures on $(X, \mathcal{F})$, i.e., monotone measures satisfying $\|\mu\| = 1$, are also called capacity, depending on the context.

For a fixed measurable space $(X, \mathcal{F})$, i.e., a non-empty set X equipped with a σ-algebra $\mathcal{F}$, recall that a function $f : X \to [0, \infty]$ is called $\mathcal{F}$-measurable if, for each $B \in \mathcal{B}([0, \infty])$, the σ-algebra of Borel subsets of $[0,\infty]$, the preimage $f^{-1}(B)$ is an element of $\mathcal{F}$.

Definition 2.5. Let $(X, \mathcal{F}, \mu)$ be a monotone measure space and $f : X \to [0, \infty]$ be an $\mathcal{F}-$measurable function. The Choquet expectation (integral) of f with respect to (w.r.t.) real monotone measure μ is defined by

$$\mathbb{E}_C^{\mu}[f] = \int_0^{\infty} \mu(X \cap \{f \geqslant y\}) dy,$$

where the integral on the right-hand side is the (improper) Riemann integral.

Mesiar [24] developed a type of integral, the so-called Choquet-like integral, which generalizes the concepts of some well-known integrals, including the Sugeno integral and the Choquet integral.

There are two classes of Choquet-like integral: the Choquet-like integral (denoted by $\mathbb{E}_{Cl,g}^{\mu}$) based on a g-addition and a g-multiplication and the Choquet-like integral based on $\vee$ and a corresponding pseudo-multiplication $\odot$. Observe that for a $\oplus$-measure, Choquet-like integrals coincide with the corresponding pseudo-additive integrals.

Theorem 2.6 24. *Let $\oplus$ and $\odot$ be generated by a generator g. Then the Choquet-like expectation of a measurable function $f : X \to [0, \infty]$ w.r.t. a real monotone measure μ can be represented as*

$$\mathbb{E}_{Cl,g}^{\mu}[f] = g^{-1}\left(\mathbb{E}_C^{g(\mu)}[g(f)]\right) = g^{-1}\left(\int_0^{\infty} g(\mu(X \cap \{g(f) \geqslant y\}))dy\right).$$

Notice that we sometimes call this kind of Choquet-like integral a g-Choquet integral ($g - C-$integral for short). It is plain that the $g - C-$ integral is the original Choquet integral (expectation) whenever $g = i$ (the identity mapping).

Theorem 2.7 24. *Let $\odot$ be a pseudo-multiplication corresponding to $\vee$ and fulfilling (M1)–(M7). Then the Choquet-like integral (so-called $\odot - \mathbb{S}_{\mu}$ integral) of a measurable function $f : X \to [0, \infty]$ w.r.t. a real monotone measure μ can be represented as*

$$\odot - \mathbb{S}_{\mu}[f] = \sup_{a \in [0,\infty]} a \odot \mu(X \cap \{f \geqslant a\}).$$

It is plain that the $\odot - \mathbb{S}_{\mu}$ integral is the original Sugeno integral whenever $\odot = \wedge$[36].

Restricting now to the unit interval [0,1] we shall consider the measurable function $f : X \to [0, 1]$ with $\|\mu\| = 1$. Observe that, in this case, we have the restriction of the pseudo-multiplication $\odot$ to $[0, 1]^2$ (called a semicopula or a conjunctor, i.e., a binary operation $\circledast : [0, 1]^2 \to [0, 1]$ which is non-decreasing in both components, has 1 as neutral element and satisfies $a \circledast b \leqslant \min(a, b)$ for all $(a, b) \in [0, 1]^2$, see [11,16]). In a special case, for a fixed strict t-norm T, the corresponding $\odot - \mathbb{S}_{\mu}$ integral is the so-called Sugeno-Weber integral [43]. If $\odot$ is the standard product, then the Shilkret integral [34] can be recognized.

The $\odot - \mathbb{S}_{\mu}$ integral on the [0,1] scale related to the semicopula $\circledast$ is given by

$$\circledast - \mathbb{S}_{\mu}[f] = \sup_{a \in [0,1]} a \circledast \mu(X \cap \{f \geqslant a\}).$$

This type of integral was called seminormed integral in [35].

Recently, Girotto and Holzer [19] proved the following Chebyshev type inequality for Choquet integral (expectation).

Theorem 2.8. *Let $(X, \mathcal{F})$ be a measurable space and Y, Z be $\mathcal{F}$-Borel measurable functions. If Y, Z are comonotone and two real-valued functions, then the following version of Chebyshev inequality:*

$$\|\mu\| \mathbb{E}_C^{\mu}[YZ] \geqslant \mathbb{E}_C^{\mu}[Y] \mathbb{E}_C^{\mu}[Z], \tag{2.1}$$

holds for any real monotone set function μ on $\mathcal{F}$, when $Y, Z \geqslant 0$, and for any real (finitely) additive measure μ on $\mathcal{F}$, when Y, Z are Choquet integrable.

Before stating our main result, we need a definition.

Definition 2.9. Let $A, B : [0,\infty)^2 \to [0,\infty)$ be two binary operations. Then A dominates B (or B is dominated by A), denoted by $A \gg B$, if

$$A(B(a,b),B(c,d)) \geqslant B(A(a,c),A(b,d))$$

holds for any $a,b,c,d \in [0,\infty)$.

Now, our results can be stated as follows.

3. Main results

The aim of this section is to show the Chebyshev type inequality for Choquet-like expectation.

Theorem 3.1. *Let $u,\ v : X \to [0,\infty)$ be two comonotone functions. Then the inequality*

$$\|\mu\| \odot (\mathbb{E}^{\mu}_{Cl,g}[u \odot v]) \geqslant (\mathbb{E}^{\mu}_{Cl,g}[u]) \odot (\mathbb{E}^{\mu}_{Cl,g}[v]) \tag{3.1}$$

holds for the g-Choquet integral if the generator g is a real-valued function.

Proof. Using the Chebyshev integral inequality for Choquet integral (2.1), we have

$$\begin{aligned}\|\mu\| \odot (\mathbb{E}^{\mu}_{Cl,g}[u \odot v]) &= g^{-1}(g(\|\mu\|)\cdot \mathbb{E}^{g(\mu)}_{C}[g(u)g(v)]) \geqslant g^{-1}[\left(\mathbb{E}^{g(\mu)}_{C}[g(u)]\right)\cdot\left(\mathbb{E}^{g(\mu)}_{C}[g(v)]\right)]\\ &= g^{-1}[g(g^{-1}(\mathbb{E}^{g(\mu)}_{C}[g(u)]))\cdot g(g^{-1}(\mathbb{E}^{g(\mu)}_{C}[g(v)]))] = g^{-1}(g(\mathbb{E}^{\mu}_{Cl,g}[u])\cdot g(\mathbb{E}^{\mu}_{Cl,g}[v]))\\ &= (\mathbb{E}^{\mu}_{Cl,g}[u]) \odot (\mathbb{E}^{\mu}_{Cl,g}[v]).\end{aligned}$$

This completes the proof. □

Theorem 3.2. *Let $u, v : X \to [0,\infty)$ be two comonotone functions and $\star : [0,\infty)^2 \to [0,\infty)$ be continuous and non-decreasing in both arguments. If $\odot$ is a pseudo-multiplication (with neutral element e) corresponding to $\vee$ satisfying*

$$(a \star b) \odot c \geqslant [(a \odot c)\star b] \vee [a \star (b \odot c)], \tag{3.3}$$

then the inequality

$$\odot - \mathbb{S}_{\mu}[\frac{u}{\|\mu\|} \star v] \geqslant \odot - \mathbb{S}_{\mu}[\frac{u}{\|\mu\|}]\star(\odot - \mathbb{S}_{\mu}[v]) \tag{3.4}$$

holds for the $\odot - \mathbb{S}_{\mu}$ integral and any real monotone set function μ such that $a \odot \|\mu\| \leqq a$ for all a and $\odot - \mathbb{S}_{\mu}[\frac{u}{\|\mu\|}]$, $\odot - \mathbb{S}_{\mu}[v]$ are finite.

Proof. Let $\odot - \mathbb{S}_{\mu}[v] = q < \infty, \odot - \mathbb{S}_{\mu}[\frac{u}{\|\mu\|}] = \frac{p}{\|\mu\|} < \infty$. Then, for any $\varepsilon > 0$, there exist q_{ε} and $\frac{p_{\varepsilon}}{\|\mu\|}$ such that $M_2 = \mu(X \cap \{v \geqslant q_{\varepsilon}\}),\ M_1 = \mu\left(X \cap \left\{\frac{u}{\|\mu\|} \geqslant \frac{p_{\varepsilon}}{\|\mu\|}\right\}\right)$ where $(q_{\varepsilon} \odot M_2) \geqslant q - \varepsilon$ and $(\frac{p_{\varepsilon}}{\|\mu\|} \odot M_1) \geqslant \frac{p}{\|\mu\|} - \varepsilon$. The comonotonicity of u,v and the monotonicity of $\star$ and the fact of $\{u \geqslant p_{\varepsilon}\} \cap \{v \geqslant q_{\varepsilon}\} \subset \left\{\frac{u}{\|\mu\|} \star v \geqslant \frac{p_{\varepsilon}}{\|\mu\|} \star q_{\varepsilon}\right\}$imply that

$$\mu\left(X \cap \left\{\frac{u}{\|\mu\|} \star v \geqslant \frac{p_{\varepsilon}}{\|\mu\|} \star q_{\varepsilon}\right\}\right) \geqslant M_1 \wedge M_2.$$

Therefore,

$$\begin{aligned}\sup_{a \in [0,\infty]}\left(a \odot \mu(X \cap \{(\frac{u}{\|\mu\|} \star v) \geqslant a)\}\right) &\geqslant \left(\frac{p_{\varepsilon}}{\|\mu\|} \star q_{\varepsilon}\right) \odot \mu\left(X \cap \left\{(\frac{u}{\|\mu\|} \star v) \geqslant (\frac{p_{\varepsilon}}{\|\mu\|} \star q_{\varepsilon})\right\}\right) \geqslant \left(\frac{p_{\varepsilon}}{\|\mu\|} \star q_{\varepsilon}\right) \odot (M_1 \wedge M_2)\\ &= [\left(\frac{p_{\varepsilon}}{\|\mu\|} \star q_{\varepsilon}\right) \odot M_1] \wedge [\left(\frac{p_{\varepsilon}}{\|\mu\|} \star q_{\varepsilon}\right) \odot M_2]\\ &\geqslant \left[\left(\frac{p_{\varepsilon}}{\|\mu\|} \odot M_1\right) \star q_{\varepsilon}\right] \wedge \left[\frac{p_{\varepsilon}}{\|\mu\|} \star (q_{\varepsilon} \odot M_2)\right] \geqslant \left[\left(\frac{p}{\|\mu\|} - \varepsilon\right) \star q_{\varepsilon}\right] \wedge \left[\frac{p_{\varepsilon}}{\|\mu\|} \star (q - \varepsilon)\right]\\ &\geqslant \left(\frac{p}{\|\mu\|} - \varepsilon\right) \star (q - \varepsilon).\end{aligned}$$

Whence $\odot - \mathbb{S}_{\mu}\left[\frac{u}{\|\mu\|} \star v\right] \geqslant \frac{p}{\|\mu\|} \star q$ follows from the continuity of $\star$ and the arbitrariness of ε. □

Remark 3.3. If $(x \star e) \vee (e \star x) \leqslant x$ for any $x \in [0, \infty)$ and $\odot$ (with neutral element e) dominates $\star$, then (3.3) holds readily. Indeed,

$$(a \star b) \odot c \geqslant [(a \star b) \odot (c \star e)] \geqslant (a \odot e) \star (b \odot c) = a \star (b \odot c),$$

and $(a \star b) \odot c \geqslant (a \odot c) \star b$ follows similarly

If $\odot = \wedge$, i.e., for the Sugeno integral, and $\star$ bounded from above by the minimum and minimum dominates $\star$, then we have a general version of Chebyshev inequality for the Sugeno integral on an arbitrary real monotone measure μ based on a product-like operation $\star$.

Notice that when working on [0,1] in Theorem 3.2, we mostly deal with $\|\mu\| = e = 1$, then $\odot = \circledast$ is semicopula (t-seminorm). Then we get the Chebyshev type inequality for seminormed fuzzy integrals [28].

Corollary 3.4. *Let $(X, \mathcal{F})$ be a measurable space and $u, v : X \to$ [0,1] be two comonotone measurable functions. Let $\star : [0,1]^2 \to [0,1]$ be continuous and non-decreasing in both arguments. If semicopula $\circledast$satisfies*

$$[(a \star b) \circledast (c)] \geqslant [(a \circledast c) \star b] \vee [a \star (b \circledast c)], \tag{3.5}$$

then the inequality

$$\circledast - \mathbb{S}_\mu[u \star v] \geqslant \left(\circledast - \mathbb{S}_\mu[u]\right) \star \left(\circledast - \mathbb{S}_\mu[v]\right) \tag{3.6}$$

holds for any capacity μ.

Remark 3.5. We can use an example in [28] to show that the condition of $[(a \star b) \circledast c] \geqslant [(a \circledast c) \star b] \vee [a \star (b \circledast c)]$ in Corollary 3.4 (thus the condition (3.3) in Theorem 3.2) cannot be abandoned, and so we omit it here.

Remark 3.6. Our results generalize those of Agahi et al. [1] as the pseudo-integrals discussed there are special instances of our Choquet-like expectations (measure in their case is rather special, necessarily pseudo-additive, while we work with a monotone measure).

4. Concluding remarks

We have proposed new versions of Chebyshev inequality for different kinds of Choquet-like integrals, thus generalizing some results already known from the literature for the Choquet and for the Sugeno integrals. Note that the area of integral inequalities is a living area important for applications, especially when approximations have to be considered, with several fresh generalizations either in the classical Lebesgue integral setting, see for example [38], or the setting of Sugeno integrals [3]. In our future work we aim to focus on the study of inequalities considering universal integrals recently proposed in [23], focusing especially on copulas-based universal integrals (covering both the Choquet and Sugeno integrals as particular cases). Recently, Agahi et al. [4] have proved some probability inequalities, including Hölder's inequality, Minkowski's inequality, Markov's inequality and Fatou's lemma for Choquet-like expectation based on a monotone measure in a rather general form.

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